Theory

of

Coupled Rooms

For: Internal only

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1.00 Object

1.1. The object of this document is present the theory and calculations to estimate the reverberant sound pressure level in a room due to a sound source operating in an adjacent ‘coupled’ room.

2.00 Scope

2.1. The scope of this document is limited to a:

1. Assumptions;
2. Theory;
3. Discussion.

3.00 Assumptions

3.1. For the theory to be valid the following assumptions must be made:

1. A truly diffuse sound field is generated in each room;
2. Each room can be considered statistically classical;
3. Steady state conditions;
4. Reverberation follows steady state conditions;

4.00 Theory

4.1. This theory is based on chapter II.3 of “Principles and Applications of Room Acoustics, Volume 1” by Cremer et. al.

4.2. Figure 1 shows the general situation where Room 1 is coupled to Room 2 through an open coupling area S_c.

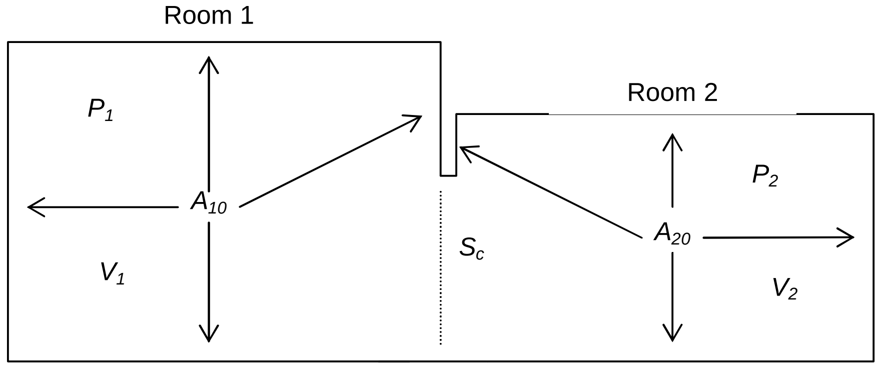

Figure 1

where:

S_c is the cross sectional area of the coupling between Room 1 and Room 2 (m^2),
P_1 is the acoustic power of a noise source operated in Room 1 (W),
V_1 is the volume of Room 1 (m^3),
A_{10} is the absorption in Room 1 excluding the area S_c (m^2),
P_2 is the acoustic power of a noise source operated in Room 2 (W),
V_2 is the volume of Room 2 (m^3),
A_{20} is the absorption in Room 2 excluding the area S_c (m^2),

Steady state sound pressure level

4.3. If we first turn off source P_2 and consider only the energy densities related to source P_1, we can say the following:

4.4. The sound power absorbed in each Room is given by:

$$\frac{E_{11}c}{4}A_{10} \text{ for Room 1 and } \frac{E_{21}c}{4}A_{20} \text{ for Room 2} \quad [1]$$

where E_{11} and E_{21} are the average energy densities due to source P_1 in Room 1 and Room 2 respectively and c is the speed of sound in air.

4.5. The power transferred between rooms is given by:

$$\text{Room 1 > Room 2 : } \frac{E_{11}c}{4}S_c \text{ and Room 2 > Room 1 : } \frac{E_{21}c}{4}S_c \quad [2]$$

4.6. Equations [1] and [2] lead to the following power balance equations:

$$P_1 - \frac{E_{11}c}{4}A_{10} - \frac{E_{11}c}{4}S_c + \frac{E_{21}c}{4}S_c = 0 \quad [3]$$

and

$$\frac{E_{11}c}{4}S_c - \frac{E_{21}c}{4}A_{20} - \frac{E_{21}c}{4}S_c = 0 \quad [4]$$

4.7. The total absorption in Room 1 and Room 2 including the coupling area is given by:

$$A_{11} = A_{10} + S_c \text{ and } A_{22} = A_{20} + S_c \text{ respectively.} \quad [5]$$

4.8. Substituting equations [5] into equations [3] and [4] leads to:

$$E_{11} = \frac{4}{c}\left(\frac{A_{22}P_1}{A_{11}A_{22} - S_c^{\ 2}}\right)\text{J/m}^3 \qquad [6]$$

and

$$E_{21} = \frac{4}{c}\left(\frac{S_c P_1}{A_{11}A_{22} - S_c^{\ 2}}\right) = E_{11}\frac{S_c}{A_{22}}\ \text{J/m}^3 \qquad [7]$$

4.9. Using the same procedure with source P_1 off and source P_2 on leads to:

$$E_{22} = \frac{4}{c}\left(\frac{A_{11}P_2}{A_{11}A_{22} - S_c^{\ 2}}\right)\text{J/m}^3 \qquad [8]$$

and

$$E_{12} = \frac{4}{c}\left(\frac{S_c P_2}{A_{11}A_{22} - S_c^{\ 2}}\right) = E_{22}\frac{S_c}{A_{11}}\ \text{J/m}^3 \qquad [9]$$

4.10. Combining equations [6] and [9] and equations [7] and [8] gives the total energy densities in each room with both P_1 and P_2 on:

$$E_{T1} = \frac{4}{c}\left(\frac{A_{22}P_1 + S_c P_2}{A_{11}A_{22} - S_c^{\ 2}}\right)\text{J/m}^3 \qquad [10]$$

and

$$E_{T2} = \frac{4}{c}\left(\frac{A_{11}P_2 + S_c P_1}{A_{11}A_{22} - S_c^{\ 2}}\right)\text{J/m}^3 \qquad [11]$$

4.11. The pressure is related to the energy density by the following equations:

$$p_{rms}^2 = E r_o c^2$$

where r_o is the density of the air. Hence,

$$L = 20\log\left(\frac{c\sqrt{E r_o}}{p_o}\right)\text{dB} \qquad [12]$$

where p_o is 20µPa.

4.12. By operating one source at a time and measuring the sound pressure levels in each room, we can find the absorption areas A_{11}, A_{10}, A_{22} and A_{20} as follows:

$$L_{11} - L_{21} = 10\log\left(\frac{E_{11}}{E_{21}}\right) = 10\log\left(\frac{A_{22}}{S_c}\right)\text{dB}$$

giving

$$A_{20} = A_{22} - S_c = S_c\left(10^{(L_{11}-L_{21})/10} - 1\right)\text{m}^2 \quad [13]$$

and

$$L_{22} - L_{12} = 10\log\left(\frac{E_{22}}{E_{12}}\right) = 10\log\left(\frac{A_{11}}{S_c}\right)\text{dB}$$

giving

$$A_{10} = A_{11} - S_c = S_c\left(10^{(L_{22}-L_{12})/10} - 1\right)\text{m}^2 \quad [14]$$

4.13. Equations [13] and [14] show that to increase the level difference L_{11}-L_{21} and L_{22}-L_{12} we must increase the absorption areas A_{22} and A_{11} respectively.

4.14. The ratios S_c/A_{11} and S_c/A_{22} are called the coupling factors k_1 and k_2 respectively. They indicate the degree of coupling from room 1 to room 2 and from room 2 to room 1. *k* values lie between 0 and 1.

4.15. The geometric mean coupling factor is given by $k = \sqrt{k_1 k_2}$, so;

$$k_1 = \frac{S_c}{A_{11}} = \frac{S_c}{A_{10} + S_c}, \qquad k_2 = \frac{S_c}{A_{22}} = \frac{S_c}{A_{20} + S_c}$$

$$k = \sqrt{k_1 k_2} \qquad [15]$$

4.16. Low *k* values indicate 'loosely' coupled rooms with large level differences, while high *k* values indicate 'tightly' coupled rooms with small level differences.

<u>Decay processes following steady state excitation</u>

4.17. Assume that only source P_1 is on and that it is switched off($P_1=0$) at time $t=0$, allowing the energy in both rooms to decay.

4.18. We replace equations [3] and [4] by the following linear differential equations:

$$\frac{cA_{11}E_{11}(t)}{4} - \frac{cS_c E_{21}(t)}{4} = -V_1 \frac{dE_{11}(t)}{dt} \qquad [16]$$

$$-\frac{cS_c E_{11}(t)}{4} + \frac{cA_{22}E_{21}(t)}{4} = -V_2 \frac{dE_{21}(t)}{dt} \quad [17]$$

where $V_{1,2}$ are the volumes of room 1 and room 2(m^3).

4.19. Assuming the reverberant decay is composed of exponential functions, we can set:

$$E_{11}(t) = E_{011}e^{-2dt}, \qquad E_{21}(t) = E_{021}e^{-2dt} \qquad [18]$$

where d is the damping constant and indicates the rate of decay of the sound pressure.

4.20. Assuming Sabine's reverberation equation holds, we have the following relation:

$$2d = \frac{6\ln(10)}{T} \qquad [19]$$

where T is Sabine's reverberation time(s).

4.21. Substituting equations [18] into [16] and [17] and setting:

$$d_1 = \frac{cA_{11}}{8V_1}, \qquad d_2 = \frac{cA_{22}}{8V_2} \qquad [20]$$

where $d_{1,2}$ are the damping constants of the uncoupled rooms including the absorption area S_c, we get for $t=0$:

$$2V_1(d_1 - d)E_{011} - 2V_1d_1k_1E_{021} = 0 \qquad [21]$$

$$-2V_2d_2k_2E_{011} + 2V_2(d_2 - d)E_{021} = 0 \qquad [22]$$

4.22. For the above equations to be valid, we must have:

$$\frac{E_{011}}{E_{021}} = \frac{d_1k_1}{(d_1 - d)} = \frac{(d_2 - d)}{d_2k_2}$$

i.e.

$$\begin{vmatrix} (d_1 - d) & -d_1 k_1 \\ -d_2 k_2 & (d_2 - d) \end{vmatrix} = 0$$

4.23. Solving this determinant leads to a quadratic in d as follows:

$$d^2 - (d_1 + d_2)d + d_1 d_2 (1 - k^2)$$

so

$$d_{I,II} = \frac{(d_1 + d_2) \pm \sqrt{(d_1 + d_2)^2 - 4 d_1 d_2 (1 - k^2)}}{2} \qquad [23]$$

4.24. Considering the extreme cases of equation [23] gives:

For $k_1 = k_2 = k = 1$ (total coupling) we have $d_I = d_1 + d_2$ and $d_{II} = 0$, i.e. the two rooms should be treated as one with a single damping constant and volume $V = V_1 + V_2$ and total absorption area $A = A_{10} + A_{20}$.

Setting $k_1 = k_2 = k = 0$ (no coupling), $d_I = d_1$ and $d_{II} = d_2$, i.e. the two rooms should be considered separate with no interaction between them.

4.25. For all intermediate values of k, the acoustics of one room has an influence on the acoustics of the other and the resulting decay processes in both rooms are dictated by d_I and d_{II}, i.e.:

$$E_{11}(t) = E_{I1} e^{-2 d_I t} + E_{II1} e^{-2 d_{II} t} \qquad [24]$$

$$E_{21}(t) = E_{I2} e^{-2 d_I t} + E_{II2} e^{-2 d_{II} t} \qquad [25]$$

where $E_{I,II}$ are the initial values of the different exponential decays.

4.26. Using equations [21] and [22] we find:

$$E_{II1} = E_{II2} \frac{k_1}{1 - d_{II}/d_1} \text{ and } E_{I2} = E_{I1} \frac{k_2}{1 - d_I/d_2} \qquad [26]$$

4.27. We find E_{I1} and E_{II2} by substituting equations [26] into equations [24] and [25] and setting $t=0$:

$$E_{I1} = \frac{E_{011} - E_{021} k_1 / (1 - d_{II}/d_1)}{1 - k^2 / ((1 - d_I/d_1)(1 - d_{II}/d_1))} \qquad [27]$$

and

$$E_{II2} = \frac{E_{021} - E_{011}\,k_2/(1-d_I/d_2)}{1-k^2/((1-d_I/d_1)(1-d_{II}/d_1))} \qquad [28]$$

where E_{011} and E_{021} is given by equations [6] and [7] for steady state conditions.

4.28. Using the same procedure, we can find the decay processes when a source is operated in room 2.

4.29. Reverberant decays are generally expressed in dB levels, and so:

$$L(t) = 10\log\left(\frac{E(t)}{E_0}\right)\text{dB} \qquad [29]$$

where E_o is the energy density at time $t=0$.

4.30. We can modify all of the above equations for the case of coupling through a partition:

$$S_c \Rightarrow t_c S_c, \qquad A_{11} = A_{10} + S_c a_{c1} \quad \text{and} \quad A_{22} = A_{20} + S_c a_{c2}$$

where:

t_c is the transmission coefficient of the partition, assumed the same in both directions.

a_{c1} and a_{c2} are the absorption coefficients of the partition on the side of room 1 and room 2 respectively.

5.00 Discussion

5.1. Rooms are often assumed acoustically isolated from adjacent rooms when evaluating their acoustic properties. In some circumstances, this can be a very dangerous assumption.

5.2. The easiest way to show this is by taking an example. Assume we have two rooms with the following parameters:

$V_1=300m^3$, $V_2=800m^3$ and an open coupling area between them of $S_c=22m^2$, not unusual in public buildings.

5.3. The reverberation time in room 1 is said to be excessive and so is measured with a view to increasing the absorption, and so decreasing the RT to a value of 1.6secs.

5.4. Further, the reverberation time is to be evaluated over a decay of –5 to –35dB.

5.5. Room 1 is measured and the RT(-5,-35) is found to be 2.9secs. Using Sabine's equation, we find that to reduce the RT to 1.6secs requires an additional 13.5m^2 of absorption. We add this to room 1 and find that the RT(-5,-35) has only reduced to 2.3secs.

5.6. The RT(-5,-35) of room 2 was later also found to be 2.9secs.

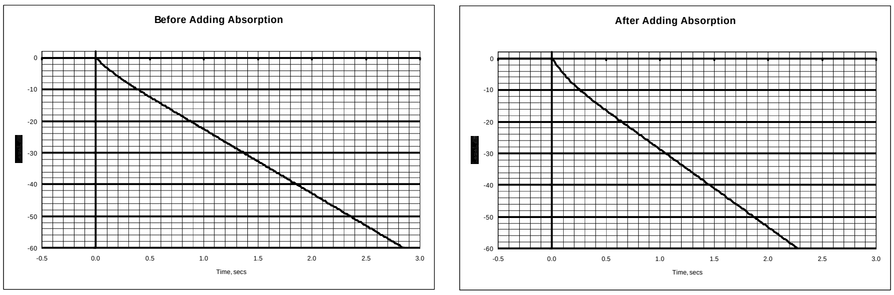

Figure 1

5.7. Using equation [24], figure 1 shows the expected decays for room 1 before and after the absorption is added.

5.8. What has happened is that after an initial time period, the long RT of room 2 has dominated the decay process.

5.9. Figure 1 shows that careful attention should be given to measured multi decays as they not only indicate uneven absorption, but also show the effects of coupling. The range over which the RT is evaluated should also be given consideration.

5.10. Figure 2 shows the decay after adding 75m^2 of absorption to room 1. The RT(-5,-35) is now 1.6secs and the double decay is evident.

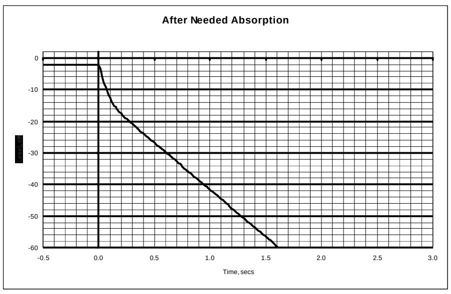

Figure 2

5.11. The safest way to determine the likely effects of coupling, is to find the coupling factors by measuring the level differences and using equations [13] and [14].

5.12. Lets now assume that we have a bar and a restaurant coupled together via an open staircase.

5.13. Complaints have been received from people in the restaurant that the noise from customers in the bar is too high.

5.14. The following are the parameters of the two rooms:

Room 1: Bar, Room 2: Restaurant	
Volume (V1) :	$300m^3$
Total surface area (S1) :	$269m^2$
Power level (P1) :	94dB (bar customers)
Volume (V2) :	$250m^3$
Total surface area (S2) :	$238m^2$
Power level (P2)	85dB (restaurant customers)
Coupling area (Sc) :	$16m^2$

5.15. Sound pressure level differences between the rooms were measured:

$L_{22} – L_{12}$ = 4dB, $L_{11} – L_{21}$ = 6dB

5.16. Using the above data and applying the formulae, we calculate that the current signal-to-noise ratio in the restaurant is –5dB (i.e. speech level in restaurant less noise from bar).

5.17. To increase the S/N ratio in the restaurant, we must increase the absorption in the bar. We can calculate the mathematical maximum achievable S/N ratio by setting the absorption coefficient of the bar surfaces to unity. The result is:

Maximum achievable S/N(restaurant) = 3.3dB

5.18. Of course, this is purely theoretical since diffuse conditions would no longer occur, and we would most likely be considering the effects of direct level propagation.